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Effects of a vertical discontinuity in a porous medium on a plane convection plume at high Rayleigh numbers

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Abstract—Standard methods exist for obtaining higher-order boundary-layer solutions at high Rayleigh numbers for convection flows in fluid-saturated porous media. The properties of the medium are usually considered uniform, however, and this is rarely the case in physical situations. We extend the theory to the case where the medium consists of two half-planes each with different permeabilities and diffusivities. A plane convection plume at the interface between the different media will be affected by this discontinuity, and we obtain results for the centreline velocity and concentration in terms of the ratios of permeabilities and diffusivities. Crossflow only occurs at second order, and many unexpected symmetries of the flow are found.

These results should be of practical interest in tracer tests to identify heterogeneities in porous rocks.

INTRODUCTION

THE BOUNDARY-LAYER formulation of Darcy's law and the energy equation has been used with considerable success in a number of aspects of diffusion in fluid saturated porous media. An analysis of natural convection from a heated impermeable surface embedded in the medium has been used by Cheng and Minkowycz [1] to model heating of groundwater in an aquafier by a dike. Several other boundary layer natural and mixed convection flows are described in detail by Cheng [2].

In order to improve the estimates based on boundary-layer theory at smaller values of the Rayleigh number (and therefore shorter downstream distances), Cheng and Chang [3] use singular perturbation theory for these problems. This gives first-order corrections to the boundary layer solutions for vertical flows in which a given power law temperature dependence in x is prescribed on a downstream surface. The results are valid therefore only when the temperature is known, such as isothermal, and Joshi and Gebhart [4] use the method of matched asymptotic expansions [5] to solve the general case where only the zero-order surface temperature dependence is known. Three types of heating conditions were considered, including the line heat plume, in which a two-dimensional source provides a constant heat flux in a fluid saturated porous medium. The method allows consistent approximations up to second order.

All the porous media considered above have been homogeneous and uniform, however, and this is rarely the case in natural rock systems. Indeed, tracer tests rely on distortions in the diffusion patterns of solutes to identify heterogeneities in the porous medium. Some measure of how much asymmetry is caused in the flow pattern by sharp changes in rock type is therefore useful and it is this problem we now explore. Specifically, we consider a line source plume at the interface between two porous media, each with different permeabilities and diffusivities. This is probably the simplest model problem to investigate, since the zero-order solution for a uniform medium is known analytically [6]. The boundary conditions at the interface yield linear differential eigenvalue problems at each order to solve for the crossflow between media, the centreline concentration and the centreline velocity. By following the method of Joshi and Gebhart [4], we find that crossflow only occurs at second order, while simple relationships exist between the solution to the present problem and the uniform medium case at zero and first order.

FORMULATION

The governing 2-D Boussinesq natural convection equations in a fluid-saturated porous medium [6] are: the continuity equation

$$u = \psi_y, \quad v = -\psi_x, \tag{1}$$

Darcy's law

$$\nabla^2\psi = \frac{K}{\mu}\rho_r g\beta\frac{\partial s}{\partial y} \tag{2}$$

and the energy equation

$$\psi_y\frac{\partial s}{\partial x} - \psi_x\frac{\partial s}{\partial y} = \kappa\nabla^2 s. \tag{3}$$

The boundary conditions at an interface between two different media along $y = 0$ are:

continuity of velocity

$$\left[\frac{\partial\psi}{\partial y}\right] = 0, \quad \left[\frac{\partial\psi}{\partial x}\right] = 0 \tag{4a, b}$$

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NOMENCLATURE

$A_i(x)$	Airy function
A_1, B_1, C_1	constants in equations (38) and (40)
c_1	ratio of diffusivities in the media, κ_+/κ_-
c_2	ratio of permeabilities of the media, K_+/K_-
$\mathrm{d}(x)$	downstream zero-order concentration decay, $Nx^{-1/3}$
f	non-dimensional stream function, $\psi/(\kappa Ra_x^{1/2})$
$G(x)$	function in equation (20)
g	gravitational acceleration
h.o.t.	higher-order terms
K	permeability of the porous medium
N	constant in the equation for $d(x)$
P	$[\rho g\beta KN/\mu\kappa]^{1/2}$
Q	source flux of the line plume
r	radial cylindrical polar coordinate, $(x^2+y^2)^{1/2}$
Ra_x	local Rayleigh number, $P^2x^{-1/3}$
s	concentration
u	x-component of velocity
v	y-component of velocity
x	vertical coordinate
y	horizontal coordinate.

Greek symbols

κ	diffusivity of the porous medium
β	density fraction of the solute
δ	boundary-layer thickness, $x^{2/3}P^{-1}$
ε	perturbation parameter, $Ra_x^{-1/2}$
μ	viscosity of the fluid
ρ	density of the fluid
η	nondimensional horizontal coordinate, y/δ
ψ	stream function
θ	angular cylindrical polar coordinate, $\tan\theta = y/x$.

Subscripts

r	reference value
0	condition when $\varepsilon = 0$
1	first-order correction
2	second-order correction
+	value for $y > 0$
−	value for $y < 0$ (no subscript implies the value for $y > 0$).

Operators

[ ]	$[f] = f(0+)-f(0-)$.

and continuity of flux of solute

$$-[s\psi_x] = [\kappa s_y]+\int_{-\infty}^{\infty}\kappa s_{xx}\,\mathrm{d}y \quad (x>0). \tag{4c}$$

This last equation is obtained from the energy equation by integrating along a horizontal line y = constant, and using the fact that for a line plume, since the source is providing a constant flux of solute [6]

$$\int_{-\infty}^{\infty} s\psi_y\,\mathrm{d}y = Q = \text{constant}. \tag{5}$$

Finally, there are the conditions at infinity:

$$s\to 0,\quad u\to 0 \quad \text{as} \quad r\to\infty,\quad \theta\neq 0. \tag{6}$$

Within the inner boundary-layer region, where $y = O(\delta)$ and $x \gg P^{-1}$, we expand the streamfunction and concentration as below. Here δ is the boundary-layer thickness and P^{-1} is the source region size, and eigenfunctions may also have to be added

$$\psi(x,y) = \kappa Ra_x^{1/2}\{f_0(\eta)+\varepsilon f_1(\eta)+\varepsilon^2 f_2(\eta)+\text{h.o.t.}\} \tag{7}$$

$$s(x,y) = \mathrm{d}(x)\{\phi_0(\eta)+\varepsilon\phi_1(\eta)+\varepsilon^2\phi_2(\eta)+\text{h.o.t.}\}. \tag{8}$$

The perturbation parameter is $\varepsilon = Ra_x^{-1/2}$ [3] and the Rayleigh number [6] is given by

$$Ra_x = \rho g\beta KNx^{2/3}/\mu\kappa. \tag{9}$$

Now, in equations (7) and (8) it is understood that one expansion, with the subscript + will hold for $\eta > 0$, while another, bearing the subscript −, will hold for $\eta < 0$. The advantage of formulating the problem in this way is that at each order one differential equation holds for all η, but with a jump condition at $\eta = 0$ relating $f_{j+}(\eta)$ with $f_{j-}(\eta)$. In this way, the solution of the boundary-value problem at each order is a complicated eigenvalue problem.

We assume different values of K and κ in the two half-planes, and N will also be different, from the solution of the zero-order problems. The other parameters, however, will all be constant.

In the outer region, far from the boundary layer and where $\theta \neq 0$, $r \gg P^{-1}$, the following expansion holds:

$$\psi = \tilde{\psi}_0+\tilde{\psi}_1+\tilde{\psi}_2+\cdots, \tag{10}$$

$$s = 0. \tag{11}$$

Furthermore, in this outer region the flow is irrotational.

The quantities of prime interest in this calculation are the centreline velocities and concentrations, given by $f_j'(0)$ and $\phi_j(0)$, and also the crossflow between the two media, measured by

$$\psi_x(x,0) = \tfrac{1}{3}\kappa P^3 Ra_x^{-1} f_0(0)-\tfrac{1}{3}\kappa P^3 Ra_x^{-2} f_2(0)+\text{h.o.t.} \tag{12}$$

ANALYSIS

The zero-order boundary-layer problem for f_0 and ϕ_0 in equations (7) and (8) is given in refs. [2] and [6] as

$$f'_0-\phi_0 = 0, \quad (13)$$

$$\phi''_0+\tfrac{1}{3}(f_0\phi_0)' = 0. \quad (14)$$

The boundary conditions for this case are, however,

$$[\kappa P^2 f''_0] = [\kappa P f_0] = [\kappa N P \phi'_0] = 0 \quad (15a–c)$$

from equation (4), and

$$\phi_0 \to 0,\ \ f'_0 \to 0 \quad (\eta \to \infty) \quad (16a, b)$$

from equation (6).

Eliminating ϕ_0 between equations (13) and (14) gives

$$f'_0+\tfrac{1}{6}f_0^2 = a\eta-b, \quad (17)$$

with the solution

$$f_0 = 6\lambda\frac{A'_i(\lambda\eta-\mu)}{A_i(\lambda\eta-\mu)}, \quad \lambda = (a/6)^{1/3}, \quad \mu = b(6a^2)^{-2/3}. \quad (18)$$

Here the function $A_i(x)$ is the usual Airy function, and the constants λ and μ must now be found by applying the conditions (15) at $\eta = 0$. This gives

$$[\kappa P\lambda G(\mu)] = 0 \quad (19a)$$

$$[\lambda^2\kappa P^2(\mu+2G^2(\mu))] = 0 \quad (19b)$$

$$[\lambda^3\kappa NP(\tfrac{1}{2}+2G^3(\mu)+\mu G(\mu))] = 0 \quad (19c)$$

where

$$G(x) = A'_i(x)/A_i(x). \quad (20)$$

An obvious solution to the system of equations (19) is $\lambda_+ = \lambda_- = 0$. Apart from this, it is important to note that for (18) to be a valid solution it must not contain singularities for any $\eta > 0$. In particular, any eigenvalues μ must be greater than the largest root of $A_i(x)$. An extensive numerical search for solutions to the system (19) for a variety of values of $c_1 = \kappa_+/\kappa_-$ and $c_2 = K_+/K_-$ failed to find any values of μ satisfying this condition. Some values are given in Table 1 for various values of c_2 and $c_1 = 1$ (which was found to give the largest values of μ).

We conclude that on the basis of numerical evidence, the only solution to the eigenvalue problem (13)–(16) with no singularities has $\lambda = 0$. Then,

$$f_{0+} = f_{0-} = \sqrt{6}\tanh[\eta/\sqrt{6}], \quad (21a)$$

$$\phi_{0+} = \phi_{0-} = \operatorname{sech}^2[\eta/\sqrt{6}], \quad (21b)$$

Table 1. Solutions to equations (19a–c), with $c_1 = 1$

c_2	$-\mu_+$	$-\mu_-$
1.1	3.6717	2.8198
1.5	3.4506	3.0455
5.0	3.3193	3.1771
10.0	3.3066	3.1898
10^4	3.2962	3.2002

with the scale constants N given by

$$K_+N_+ = K_-N_-, \quad (22)$$

and, by using equation (21) in the definition (5),

$$\tfrac{1}{2}\sqrt{\tfrac{3}{2}}Q = \left\{\left(\frac{\kappa_+}{K_+}\right)^{1/2}+\left(\frac{\kappa_-}{K_-}\right)^{1/2}\right\}\left(\frac{\rho g\beta}{\mu}\right)^{1/2}K_+N_+. \quad (23)$$

We now match this inner solution to an outer solution $\tilde{\psi}_0+\tilde{\psi}_1$, which satisfies

$$\nabla^2\tilde{\psi}_1 = 0 = \nabla^2\tilde{\psi}_0. \quad (24)$$

$\tilde{\psi}_0$ and $\nabla\tilde{\psi}_0$ continuous everywhere,

$$\tilde{\psi}_0 \to 0 \text{ as } r \to \infty. \quad (25)$$

$$\tilde{\psi}_1(\theta = +0) = \kappa_+P_+\sqrt{6}r^{1/3}, \quad \tilde{\psi}_1(\theta = -0) = \kappa_-P_-\sqrt{6}r^{1/3}. \quad (26)$$

$\tilde{\psi}_1$ and $\nabla\tilde{\psi}_1$ continuous on $\theta = \pi$,

$$\tilde{\psi}_1 = 0(r^{1/3}). \quad (27)$$

The inhomogeneous condition (26) results from matching with the boundary-layer solution (21) as $\eta \to \infty$. Equations (24) and (25) imply that $\tilde{\psi}_0 = 0$, while equations (24), (26) and (27) can be solved by a Mellin transform, to give $\tilde{\psi}_{1\pm}$ in terms of integrals. The essential feature of this solution is the behaviour as $Pr \to \infty$, to match with the inner region. We find this by using the calculus of residues, to give

$$\tilde{\psi}_{1\pm} \sim \sqrt{6}\kappa_\pm P_\pm r^{1/3}\{\cos\tfrac{1}{3}\theta - (2c_1^{\mp 1/2}-1)3^{-1/2}\sin\tfrac{1}{3}\theta\}+0(r^{-1/2}). \quad (28)$$

Expanding this about $\theta = 0$, in the matching region

$$\tilde{\psi}_{1\pm} \sim \pm\sqrt{6}\kappa_\pm Ra_x^{1/2}\left\{1-\frac{1}{3\sqrt{3}}(2c_1^{\mp 1/2}-1)\varepsilon\eta + \frac{\varepsilon^2\eta^2}{9}+\text{h.o.t.}\right\}. \quad (29)$$

The first-order inner problem is given by

$$f''_1-\phi'_1 = 0 \quad (30)$$

$$\phi''_1+(\tfrac{2}{3}f'_0)\phi_1+(\tfrac{1}{3}\phi_0)f'_1+(\tfrac{1}{3}f_0)\phi'_1 = 0 \quad (31)$$

subject to the boundary conditions (4) at the origin

$$f'_{1+}(0) = c_1^{1/2}f'_{1-}(0), \quad (32)$$

$$f''_{1+}(0) = c_2/c_1 f''_{1-}(0), \quad (33)$$

and at infinity

$$\phi_{1\pm}(\pm\infty) = 0 \quad (34a)$$

$$f_{1\pm}(\eta) \to \mp\frac{\sqrt{2}}{3}(2c_1^{\mp 1/2}-1)\eta+\begin{cases}A_1 & (\eta \to +\infty)\\ B_1 & (\eta \to -\infty).\end{cases} \quad (34b)$$

This last condition is found by matching with equation (29). The equations were integrated numerically to give a set of complementary functions, and using the boundary conditions gave a linear system of equations to solve for the unknowns $f_1(0), f'_1(0), \phi_1(0), \phi'_1(0)$. It was found numerically that in all cases

$$f'_1(0) = f_1(0) = \phi'_1(0) = 0. \quad (35)$$

This result is not at all obvious from an investigation *a priori* of the problem.

Because all solutions have this property, they reduce to a form very similar to those calculated by Joshi and Gebhart [4]. For the uniform medium case, equation (34b) becomes $f'_1(\infty) = -\sqrt{2/3}$, and by integrating equation (31) over $[0, \infty]$, $\phi_1(0) = -f'_1(\infty)$. This is why in ref. [4], $\phi_1(0) = 0.4714$. In our case,

$$f'_1(0) = 0, \quad \phi_{1\pm}(0) = \pm\frac{\sqrt{2}}{3}(2c_1^{\mp 1/2}-1), \quad (36)$$

and

$$(f_1, \phi_1) = (2c_1^{\mp 1/2}-1)(F_1, \Phi_1), \quad (37)$$

where F_1 and Φ_1 are the solution given by Joshi and Gebhart in Fig. 2 of their paper [4].

The second-order outer solution is governed by

$$\nabla^2\tilde{\psi}_2 = 0, \quad (38)$$

$$\tilde{\psi}_2(\theta = +0) = \kappa_+ A_1, \quad \tilde{\psi}_2(\theta = -0) = \kappa_- B_1. \quad (39a)$$

$$\tilde{\psi}_2 \text{ and } \nabla\tilde{\psi}_2 \text{ continuous on } \theta = \pi. \quad (39b)$$

The condition (39a) comes from matching $\tilde{\psi}_1 + \tilde{\psi}_2$ with the two-term inner expansion.

We solve this by the same Mellin transform technique as was used on the first-order problem. Again, expanding about $\theta = 0$ for $r \gg P^{-1}$ in the matching region

$$\tilde{\psi}_{2\pm} = \kappa_\pm Ra_x^{1/2}\varepsilon\begin{cases} A_1 + \varepsilon C_1\eta + \text{h.o.t.} & (\eta > 0) \\ B_1 + \varepsilon C_2\eta + \text{h.o.t.} & (\eta < 0)\end{cases} \quad (40a)$$

where

$$C_1 = -\frac{c_1^{-1}B_1 + A_1}{2\pi} \quad \text{and} \quad C_2 = -\frac{B_1 + c_1A_1}{2\pi}. \quad (40b)$$

The second-order inner problem is then obtained as

$$f''_2 + \{\tfrac{4}{9}\eta^2 f''_0 + \tfrac{2}{3}\eta f'_0 - \tfrac{2}{9}f_0\} - \phi'_2 = 0, \quad (41)$$

$$\phi''_2 + [\tfrac{4}{9}\eta^2\phi''_0 + \tfrac{14}{9}\eta\phi'_0 + \tfrac{4}{9}\phi_0] + f'_0\phi_2 + [\tfrac{2}{3}f'_1\phi_1] + \tfrac{1}{3}\phi_0 f'_2 - \tfrac{1}{3}\phi'_0 f_2 + \tfrac{1}{3}f_0\phi'_2 = 0. \quad (42)$$

The boundary conditions at this order become complicated as well. The continuity of velocity (4a,b) requires that

$$f'_{2+}(0) = c_1^{-1}f'_{2-}(0), \quad f_{2+}(0) = c_1^{-3/2}f_{2-}(0). \quad (43)$$

The flux condition at the interface (4c) gives, at $\eta = 0$,

$$\tfrac{1}{3}(c_2^{-1}-1)f_{2-} - (c_1^{3/2}c_2^{-1}\phi'_{2+} - \phi'_{2-}) = 2(2/3)^{1/2}(c_1^{3/2}c_2^{-1}-1). \quad (44)$$

Finally, by matching $\tilde{\psi}_0 + \tilde{\psi}_1 + \tilde{\psi}_2$ with the three-term inner expansion, we obtain the conditions at infinity

$$f'_{2\pm}(\eta) \to \pm(\tfrac{2}{3})^{3/2}\eta + \begin{cases} C_1 & (\eta \to +\infty) \\ C_2 & (\eta \to -\infty)\end{cases}, \quad \phi_2(\infty) = 0 \quad (45)$$

where C_1 and C_2 are given by the solution to the first-order problem, and equation (40). Finally, we note that by integrating equation (41) over $[0, \infty]$ and over $[-\infty, 0]$, the following relationship is obtained

$$f'_{2+}(0) - C_1 = \phi_{2+}(0), \quad f'_{2-}(0) - C_2 = \phi_{2-}(0). \quad (46)$$

It is hard to do any further analysis on this problem, however, and the numerical results are given in the next section. It should first be noted that in common with all matching schemes eigenfunctions satisfying the boundary conditions can always be added to the initial perturbation expansions (7) and (8). Any multiple of these eigenfunctions can be added, and the associated multiplicative constants are usually indeterminate.

For the line source plume in a uniform porous medium, it is shown in ref. [4] that the first eigenfunction only occurs at $0(\varepsilon^3)$ and so the perturbation expansions we have postulated hold good to $0(\varepsilon^2)$. For other types of convective flows, however, these eigenfunctions occur at lower orders and it becomes necessary to include logarithmic terms as done by Stewartson [7], and they will complicate the problem for the higher-order corrections. This is a reason why the line source plume was chosen to illustrate the crossflow effects at an interface between different porous media, since they are seen to occur at second order before the eigenfunctions complicate the problem.

RESULTS

The second-order linear eigenvalue problem (41)–(45) was integrated numerically by a standard shooting and matching technique which is described in detail in ref. [8]. This reduces it to a matrix inversion problem to determine the unknowns, which include $f'_2(0)$, $\phi_2(0)$ and $f_2(0)$. Then the centreline concentration is given by

$$s(x, \pm 0) = Nx^{-1/3}\left[1 \pm \frac{\sqrt{2}}{3}(2c_1^{\mp 1/2}-1)\varepsilon + \phi_{2\pm}(0)\varepsilon^2\right], \quad (47)$$

and the centreline vertical velocity is

$$u(x, 0) = \frac{\kappa}{x}Ra_x[1 + f'_2(0)\varepsilon^2]. \quad (48)$$

Finally, the crossflow up to $0(\varepsilon^2)$ is given, from equation (12), by

$$v(x, 0) = -\frac{\kappa}{3x}\varepsilon^2 f_2(0). \quad (49)$$

The computed values of $f'_2(0)$ and $\phi_2(0)$ are given in Table 2. The values of $f'_{2-}(0)$ and $\phi'_{2-}(0)$ can readily be calculated from this table from equations (43) and (44) and so are not listed. Furthermore, values corresponding to c_1^{-1} and c_2^{-1} are the same as those with c_1 and c_2 but with plus and minus signs exchanged. We find that these two parameters depend only on the value of c_1 and are quite independent of c_2. This is a surprising result, reminiscent of the first-order result in equation (35). It is not at all obvious from an analysis of the problem without computing. The values of $f_2(0)$ do, however, depend both on c_1 and c_2. For this reason they are more conveniently plotted on a graph in figure one. Altering c_2 does have a significant effect on the crossflow, therefore.

DISCUSSION

We have found solutions valid to second order for the problem of a plane plume at the interface between two fluid-saturated porous media with different permeabilities and diffusivities. Although it cannot be proved analytically that these solutions are unique, the numerical methods used do give unique answers.

What we find is a surprising amount of symmetry remaining in the problem even if the medium is inhomogeneous. At zero-order, the only solution without singularities appears to be the same one as in the homogeneous case. This conclusion is based on a numerical search for the eigenvalues of the system (19), but it does seem quite certain. The ratio of the velocities in the two media is found to be $c_1^{1/2}$, according to (22), and is therefore independent of the ratio of permeabilities. Furthermore, $f'_0(0) = 0$ and so there is no crossflow at this order.

At first order the situation is similar. A numerical investigation of the eigenvalue problem (31)–(34) yields only symmetrical solutions (35). This means that there is still no crossflow, and no correction to the centreline vertical velocity. Because of the structure of the equations, the correction to the centreline concentration is given in closed anlytical form by equation (36), and only depends on the ratio c_1. Again, the ratio c_2 seems to have little effect on the solution at this order.

It is only at second order that any crossflow occurs, and is given by equation (43) and Fig. 1. This quantity does depend on both c_1 and c_2. The centreline vertical

Table 2. Results of the second-order computations

c_1	$f'_2(0)$	$\phi_2(0)$
0.25	0.3326	0.4811
0.5	0.3111	0.4775
0.75	0.2978	0.4763
1.0	0.2823	0.4760
2.0	0.2656	0.4742
3.0	0.2559	0.4729
4.0	0.2526	0.4716
5.0	0.2533	0.4702

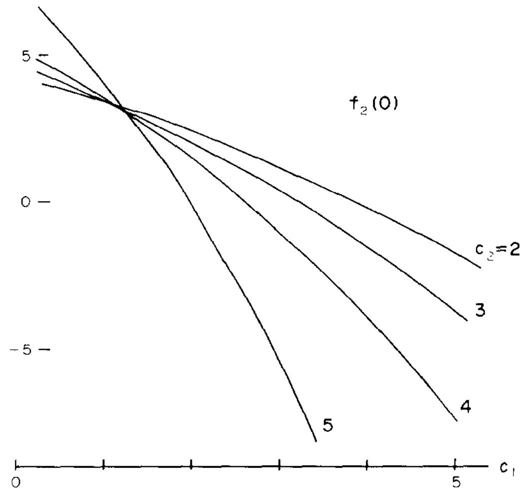

FIG. 1. Results of second-order calculations.

velocity and concentration corrections, however, are still independent of c_2 and are tabulated in Table 2.

It is difficult to proceed with the analysis past second order because of the existence of eigenfunctions. The higher-order problems then become far more complicated numerically. The results given should give very useful measures of the centreline velocities and concentrations of such flows. In particular, the crossflow result (48) can be used in tracer tests along the boundaries between rocks. Experimental data to compare with is lacking, but the results in Fig. 1 should be easy to check.

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EFFETS D'UNE DISCONTINUITE VERTICALE DANS UN MILIEU POREUX, SUR UN PANACHE DE CONVECTION PLANE A DES NOMBRES DE RAYLEIGH ELEVES

Résumé—Il existe des méthodes pour obtenir des solutions de couche limite d'ordre élevé, à nombre de Rayleigh élevés, pour des écoulements de convection dans des milieux poreux saturés de fluide. Les propriétés du milieu sont généralement considérées uniformes, néanmoins ceci est rarement le cas dans les situations réelles. On étend la théorie au cas où le milieu comprend deux demi-plans avec différentes perméabilités et diffusivités. Un panache de convection plane à l'interface entre les milieux différents est influencé par cette discontinuité et on obtient des résultats pour les vitesses et concentrations sur la ligne des centres en fonction des rapports des perméabilités et des diffusivités. Un écoulement transversal apparait seulement au second ordre et on trouve plusieurs symétries de l'écoulement. Ces résultats devraient être utilisables dans les essais par traceur pour identifier des hétérogénéités dans les roches poreuses.

DER EINFLUSS EINER VERTIKALEN DISKONTINUITÄT IN EINEM PORÖSEN MEDIUM AUF EINE EBENE KONVEKTIONSFAHNE BEI HOHEN RAYLEIGH-ZAHLEN

Zusammenfassung—Für Konvektionsströmungen in flüssigkeitsgesättigten porösen Medien existieren Standardmethoden für Grenzschichtlösungen höherer Ordnung bei hohen Rayleigh-Zahlen. Die Stoffwerte des Fluids werden gewöhnlich als konstant angenommen, was jedoch bei physikalischen Vorgängen selten der Fall ist. Wir erweitern die Theorie für den Fall eines aus zwei Halbebenen bestehenden Mediums, jede mit unterschiedlichen Permeabilitäten und Diffusionskoeffizienten. An der Grenze zwischen den beiden Stoffen wird durch diese Trennung eine ebene Konvektionsfahne hervorgerufen, und wir erhalten Ergebnisse für die Geschwindigkeit und Konzentration in der Strömungsmitte in Abhängigkeit von Permeabilitäten und Diffusionskoeffizienten. Querströmung taucht nur in Betrachtungen zweiter Ordnung auf, und es wurden manch unerwartete Strömungssymmetrien gefunden. Für Spurenuntersuchungen zur Identifizieurung von Inhomogenitäten in porösen Steinen können diese Ergebnisse von praktischem Interesse sein.

ВЛИЯНИЕ ВЕРТИКАЛЬНОЙ НЕОДНОРОДНОСТИ В ПОРИСТОЙ СРЕДЕ НА ПЛОСКУЮ КОНВЕКТИВНУЮ ФОНТАНИРУЮЩУЮ СТРУЮ ПРИ БОЛЬШИХ ЧИСЛАХ РЭЛЕЯ

Аннотация—Существуют стандартные методы получения достаточно точного решения пограничного слоя при больших числах Рэлея для конвективных течений в насыщенных жидкостью пористых средах. Свойства среды обычно принимаются однородными, хотя в реальных физических ситуациях данное предположение встречается редко. В работе теория применяется для случая, когда среда состоит из двух полуплоскостей, каждая из которых имеет свою проницаемость и температуропроводность. На плоскую конвективную фонтанирующую струю на границе между различными средами влияет указанная выше неоднородность. Получены результаты для осевых скорости и концентрации, зависящих от отношения проницаемости и температуропроводности. Поперечное течение имеет место только во втором порядке величин. Обнаружено много неожиданных свойств симметрии потока. Данные результаты представляют интерес в опытах по определению неоднородностей в пористых горных породах.